O K L A H O M A   S T A T E   U N I V E R S I T Y

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECEN 5713 Linear Systems
Spring 2009
Final Exam

Choose any four out of five problems.
Please specify which four listed below to be graded:
1)_____; 2)_____; 3)_____; 4)_____;

Name: ______________________________

E-Mail Address:_______________________________________

Problem 1:

Find the *observable* canonical form realization (in minimal order) from a continuous-time system

$$\frac{d^4y(t)}{dt^4}+3t\frac{d^3y(t)}{dt^3}+6\frac{d^2y(t)}{dt^2}-3\frac{dy(t)}{dt}+\alpha(t)y(t)=\frac{d^2u(t)}{dt^2}+2e^{-t}\frac{du(t)}{dt}+u(t).$$

Notice that the gain blocks may be *time* dependent. Show the state space representation and its corresponding simulation diagram.

Problem 2:

Find a minimal *observable* canonical form realization (i.e., its simulation diagram and state space representation) for the following MISO system described by

$$H(s) = \begin{bmatrix} \dfrac{2s+3}{s^3+4s^2+5s+2} & \dfrac{s^2+2s+2}{s^4+3s^3+3s^2+s} \end{bmatrix}.$$

Problem 3:

Let λ_i be an eigenvalue of a matrix A and let v^i be the corresponding eigenvector. Let $f(\lambda) = \sum_{k=0}^{l} \alpha_k \lambda^k$ be a polynomial with real coefficients α_k. Show that $f(\lambda_i)$ is an eigenvalue of the matrix function $f(A) = \sum_{k=0}^{l} \alpha_k A^k$ with the same coefficients α_k. Determine the eigenvector corresponding to eigenvalue $f(\lambda_i)$.

<u>Problem 4</u>:

Let

$$C = \begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix},$$

find a matrix B and existing condition, such that $e^B = C$. Is it true that for any nonsingular matrix C, there exists a matrix B such that $e^B = C$. Justify your answer.

Problem 5:

Verify that $B(t) = \Phi(t,t_0)B_0\Phi^*(t,t_0)$ is the solution of

$$\frac{d}{dt}B(t) = A(t)B(t) + B(t)A^*(t), \quad \text{with initial condition } B(t_0) = B_0,$$

where $\Phi(t,t_0)$ is the state-transition matrix of $\dot{x}(t) = A(t)x(t)$ and $\Phi^*(t,t_0)$ is the complex conjugate of $\Phi(t,t_0)$.